AP® Calculus AB
2002 Sample Student Responses

The materials included in these files are intended for use by AP teachers for course and exam preparation in the classroom; permission for any other use must be sought from the Advanced Placement Program®. Teachers may reproduce them, in whole or in part, in limited quantities, for face-to-face teaching purposes but may not mass distribute the materials, electronically or otherwise. These materials and any copies made of them may not be resold, and the copyright notices must be retained as they appear here. This permission does not apply to any third-party copyrights contained herein.

These materials were produced by Educational Testing Service® (ETS®), which develops and administers the examinations of the Advanced Placement Program for the College Board. The College Board and Educational Testing Service (ETS) are dedicated to the principle of equal opportunity, and their programs, services, and employment policies are guided by that principle.

CALCULUS AB

SECTION II, Part A

Time—45 minutes

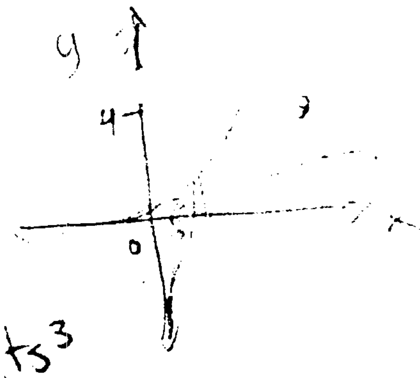
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$\int_{.5}^{1}(e^x - \ln x)\,dx = 1.223$ units^2

A2



Work for problem 1(b)

$R = 4 - \ln x$
$r = 4 - e^x$

$\pi \int_{.5}^{1} \left((4-\ln(x))^2 - (4-e^x)^2 \right) dx = 23.609 \text{ units}^3$

Work for problem 1(c)

$h(x) = f(x) - g(x)$
$= e^x - \ln x$

$h'(x) = e^x - \frac{1}{x}$

For criticals
$e^x - \frac{1}{x} = 0$

$\ln e^x = \ln \frac{1}{x}$

$x = \ln \frac{1}{x}$

$x = -\ln x$

$\ln x + x = 0$

$x = .567$

End pts: .5, 1

x	f(x)
.5	2.341
.567	2.330
1	2.718

To determine the absolute minimum and maximum I found any criticals (when h'(x) equals 0) and the end points. There was only one critical number, which occurred at x = .567. When I compared the values of each number (see chart), I found the absolute minimum value to be 2.330 and the absolute maximum value to be 2.718 (or e).

GO ON TO THE NEXT PAGE.

1 1 1 1 1 1 1 1 1 1 C1

CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$A = \int_{\frac{1}{2}}^{1} e^x - \ln x \, dx = \boxed{1.223}\ u^2$

(using fnInt)

C2

Work for problem 1(b)

$V = \pi \int_{\frac{1}{2}}^{1} (4-\ln x)^2 - (4-e^x)^2 \, dx = \boxed{23.610} \; u^3$

(using fnInt)

Work for problem 1(c)

$h'(x) = f'(x) - g'(x)$

$h'(x) = e^x - \frac{1}{x} = 0$

$h(1) = \boxed{e - \ln 1} = MAX$

e^x grows faster than $\ln x$ so on the interval $\frac{1}{2} \le x \le 1$, the greatest value of $h(x)$ will be at $x = 1$.

$h(\frac{1}{2}) = \boxed{e^{\frac{1}{2}} - \ln \frac{1}{2}} = MIN$

since e^x grows faster than $\ln x$ the min. value will be at the very beginning of the interval at $x = \frac{1}{2}$.

AP® Calculus AB
2002 Sample Student Responses

The materials included in these files are intended for use by AP teachers for course and exam preparation in the classroom; permission for any other use must be sought from the Advanced Placement Program®. Teachers may reproduce them, in whole or in part, in limited quantities, for face-to-face teaching purposes but may not mass distribute the materials, electronically or otherwise. These materials and any copies made of them may not be resold, and the copyright notices must be retained as they appear here. This permission does not apply to any third-party copyrights contained herein.

Work for problem 2(a)

$\int_9^{17} E(t)\,dt = \int_9^{17} 15600/(t^2-24t+160)\,dt = 6004.27$

≈ 6004 people

Work for problem 2(b)

$\int_9^{23} \left(\frac{15600}{t^2-24t+160}\right) dt$ = TOTAL ENTERED

$= 7275.55 \approx 7276$ PEOPLE

7276
− 6004

AFTER 5 → 1272
× \$11
\$13,992

+

BEFORE 5 6004
× \$15
\$90,060

= \$104,052 made on the given day

Work for problem 2(c)

$$H(17) = \int_9^{17} \left(\frac{15600}{t^2 - 24t + 160}\right) - \left(\frac{9890}{t^2 - 38t + 370}\right) dt = 3725$$

$$H'(17) = \left(\frac{15600}{(17^2) - 24(17) + 160}\right) - \frac{9890}{(17^2 - 38(17) + 370)}$$

$$= 380 - 760$$

$H'(17) = -380$ → This is the rate of change at 5 o'clock that PEOPLE ARE ENTERING THE PARK COMPARED TO THOSE LEAVING THE PARK MORE PEOPLE LEAVING THAN ENTERING AT $t = 17$

$H(17) = 3725$ → THIS IS THE AMOUNT OF PEOPLE INSTANTANEOUSLY AT THE PARK.

Work for problem 2(d)

$$H'(t) = \frac{15600}{(t^2 - 24t + 160)} - \frac{9890}{t^2 - 38t + 370} = 0$$

$$t = 15.7948$$

Work for problem 2(a)

$$\int_9^{17} \frac{15600}{(t^2-24t+160)}\,dt = 6004 \text{ people}$$

Work for problem 2(b)

$$15\int_9^{17} \frac{15600}{(t^2-24t+160)}\,dt + 11\int_{17}^{23} \frac{9890}{(t^2-38t+370)}\,dt$$

$$90064 + 54950$$

$$\$\,145{,}014$$

Work for problem 2(c)

$H'(t) = E(t) - L(t)$

$H'(17) = 380.4878 - 760.7692$

$H'(17) = -452.2814$

H(17) represents the number of people in the park at t=17. H'(17) represents the rate at which the population of the park is changing at t=17

Work for problem 2(d)

$E(t) - L(t) = 0$

$\frac{15600}{(t^2 - 24t + 160)} - \frac{9890}{t^2 - 38t + 370} = 0$; $t = \boxed{15.79481}$

AP® Calculus AB
2002 Sample Student Responses

These materials were produced by Educational Testing Service® (ETS®), which develops and administers the examinations of the Advanced Placement Program for the College Board. The College Board and Educational Testing Service (ETS) are dedicated to the principle of equal opportunity, and their programs, services, and employment policies are guided by that principle.

The College Board is a national nonprofit membership association dedicated to preparing, inspiring, and connecting students to college and opportunity. Founded in 1900, the association is composed of more than 4,200 schools, colleges, universities, and other educational organizations. Each year, the College Board serves over three million students and their parents, 22,000 high schools, and 3,500 colleges, through major programs and services in college admission, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT®, the PSAT/NMSQT®, and the Advanced Placement Program® (AP®). The College Board is committed to the principles of equity and excellence, and that commitment is embodied in all of its programs, services, activities, and concerns.

A1

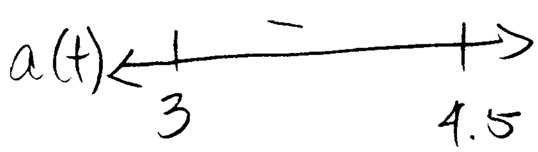
Work for problem 3(a)

$$a = \frac{dv}{dt} = \frac{\pi \cos\left(\frac{\pi t}{3}\right)}{3}$$

$$a(4) = \frac{\pi \cos\left(\frac{\pi 4}{3}\right)}{3}$$

$$a = \frac{-\pi}{6}$$

Work for problem 3(b)



$v(3) = \sin(\pi) = 0$

$v(4.5) = \sin\left(\frac{4.5\pi}{3}\right) = -1$

$a(t) < 0$

$v(t)$ is decreasing → statement I is correct

$a(t) < 0$ $v(t) \leq 0$ speed is $|v(t)|$ (no direction)

$|v(t)|$ is increasing since $v(t)$ and $a(t)$ are the same sign

↓

speed is increasing

↓

statement II is correct

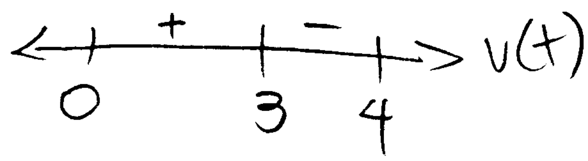
A2

Work for problem 3(c)

$v(t) = 0$

$t = 3$

$t = 0$



$x(t) = \int v(t)\,dt$

$x_{tot}(t) = \int_0^3 v(t)\,dt + \int_4^3 v(t)\,dt$

$= \frac{6}{\pi} + \frac{3}{2\pi} = \frac{15}{2\pi}$

Work for problem 3(d)

$x(t) = \int v(t)\,dt$

$\frac{-3\cos\left(\frac{t\pi}{3}\right)}{\pi} + C = x(t)$

$\frac{-3}{\pi} + C = 2$

$C = \frac{3}{\pi} + 2$

$x(4) = \frac{-3\cos\left(\frac{4\pi}{3}\right)}{\pi} + \frac{3}{\pi} + 2$

$= \frac{9}{2\pi} + 2$

Work for problem 3(a)

$v(t) = \int a(t)dt$

$v'(t) = a(t)$

$a(t) = \frac{\pi}{3}\cos(\frac{\pi}{3}t)$

$a(4) = \frac{\pi}{3}\cos(\frac{4\pi}{3})$

$\approx -.524$

Work for problem 3(b)

The function $v(t) = \sin(\frac{\pi}{3}t)$ is negative from $3<t<4.5$, so velocity is decreasing. However, the function is concave up, so acceleration must be positive. When acceleration is positive, speed is increasing, so both statements are true.

Work for problem 3(c)

$TD = \int_0^4 |v(t)|\,dt$

$= \int_0^4 |\sin(\frac{\pi}{3}t)|\,dt$

$= 2.387$ units

Work for problem 3(d)

$x(t) = \int v(t)\,dt$

$x(t) = \frac{3}{\pi}\int \sin(\frac{\pi}{3}t)\,dt \cdot \frac{\pi}{3}$

$x(t) = \frac{3}{\pi}\int \sin u\,du$

$= -\frac{3}{\pi}\cos u + C$

$= -\frac{3}{\pi}\cos(\frac{\pi}{3}t) + C$

$2 = -\frac{3}{\pi}\cos(\frac{\pi}{3}\cdot 0) + C$

$2 = -\frac{3}{\pi}(1) + C$

$2.955 = C$

$u = \frac{\pi}{3}t$

$du = \frac{\pi}{3}dt$

$x(t) = -\frac{3}{\pi}\cos(\frac{\pi}{3}t) + 2.955$

$x(4) = -\frac{3}{\pi}\cos(\frac{4\pi}{3}) + 2.955$

$x(4) \approx 3.432$

AP® Calculus AB
2002 Sample Student Responses

The materials included in these files are intended for use by AP teachers for course and exam preparation in the classroom; permission for any other use must be sought from the Advanced Placement Program®. Teachers may reproduce them, in whole or in part, in limited quantities, for face-to-face teaching purposes but may not mass distribute the materials, electronically or otherwise. These materials and any copies made of them may not be resold, and the copyright notices must be retained as they appear here. This permission does not apply to any third-party copyrights contained herein.

B1

NO CALCULATOR ALLOWED

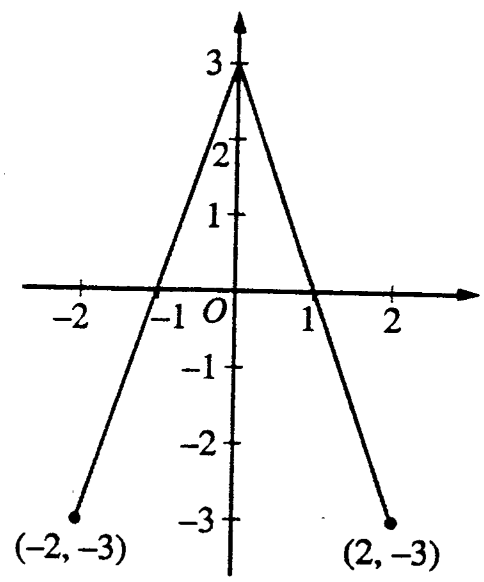
CALCULUS

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of f

Work for problem 4(a)

$g(-1) = \int_0^{-1} f(t) = -1 \cdot 3 \cdot \frac{1}{2} = \frac{-3}{2}$

$g'(-1) = f(-1) = 0$

$g''(-1) = f'(-1) = 3.$

NO CALCULATOR ALLOWED

B2

Work for problem 4(b)

$g'(x) = f(x)$

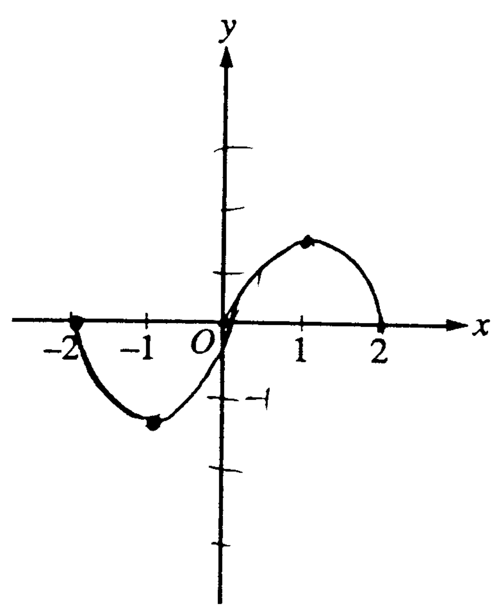
For $-1 < x < 1$, g is increasing, because $g'(x) = f(x)$ is positive for this interval.

Work for problem 4(c)

$g''(x) = f'(x)$

For $0 < x < 2$, g concaves down, because $g''(x) = f'(x)$ is negative for this interval.

Work for problem 4(d)



C1

NO CALCULATOR ALLOWED

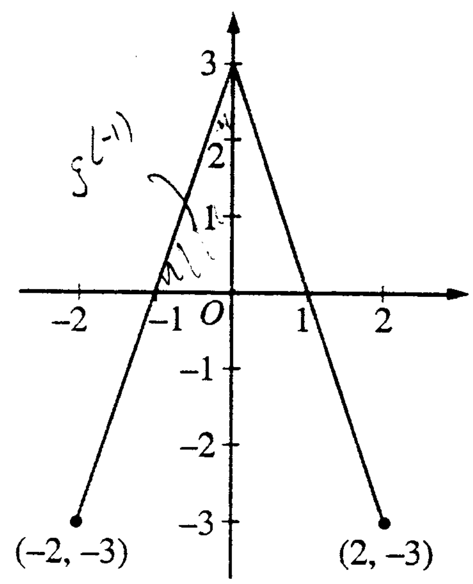
CALCULUS

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of f

Work for problem 4(a)

$g(x) = \int_0^x f(t)\,dt$

$g(-1) = \int_0^{-1} f(t)\,dt = \frac{1}{2}bh = \frac{1}{2}(1)(3) = \frac{3}{2}$

$g(-1) = \frac{3}{2}$

$g'(-1) = f(t) = 0$

$g'(-1) = 0$

$g''(-1) =$ slope from -2 to $0 = \frac{-3-3}{-2-0} = \frac{-6}{-2} = 3$

$g''(-1) = 3$

NO CALCULATOR ALLOWED

C_2

Work for problem 4(b)

$g'(x) = f(x)$

$g(x)$ increases when $f(x)$ is positive $f(x) > 0$ at $-1 < x < 1$

$g(x)$ increases at $-1 < x < 1$

Work for problem 4(c)

$g(x)$ is concave down where $g''(x)$ is negative

$g'(x) = f(x)$

$g''(x) = f'(x)$ $g''(x) = f'(x)$ is negative for all $0 < x < 2$

$f'(x) = \frac{-3-3}{2-0} = -3$

$g(x)$ is concave down for all $0 < x < 2$

Work for problem 4(d)

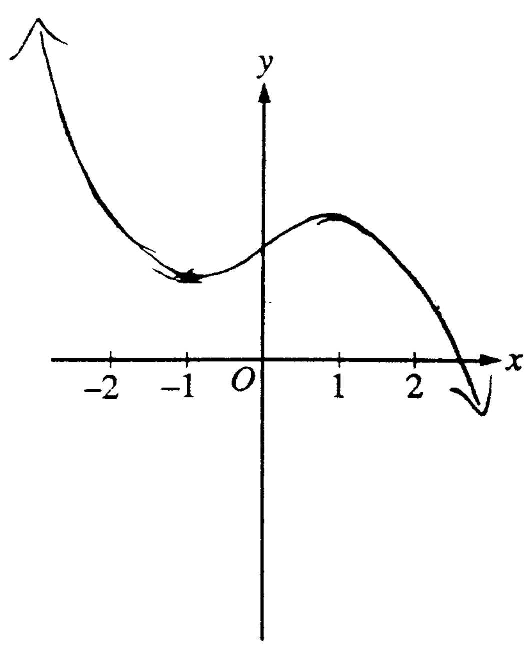
$-$ $+$ $-$ g'

-1 1

$g''(x)$

$+$ $-$

0



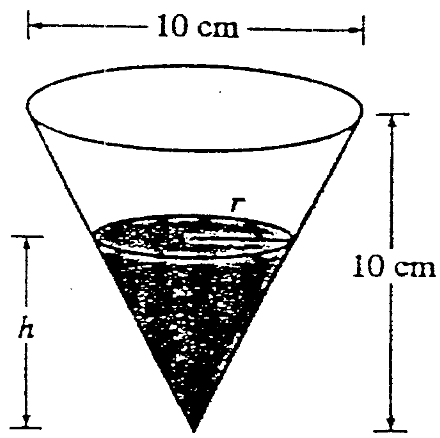
AP® Calculus AB
2002 Sample Student Responses

The materials included in these files are intended for use by AP teachers for course and exam preparation in the classroom; permission for any other use must be sought from the Advanced Placement Program®. Teachers may reproduce them, in whole or in part, in limited quantities, for face-to-face teaching purposes but may not mass distribute the materials, electronically or otherwise. These materials and any copies made of them may not be resold, and the copyright notices must be retained as they appear here. This permission does not apply to any third-party copyrights contained herein.

5 5 5 5 5 5 5 5 5 5

NO CALCULATOR ALLOWED

A1



$\frac{dh}{dt} = \frac{-3}{10}$ cm/hr

$V = \frac{1}{3}\pi r^2 h$

Work for problem 5(a)

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (2.5)^2(5)$

$V = \frac{5}{3}\pi (2.5)^2$ cm^3

$\frac{10}{5} = \frac{5}{r}$ $r = 2.5$ cm

$\frac{h}{r} = 2$

$\frac{h}{2} = r$

NO CALCULATOR ALLOWED

A2

Work for problem 5(b)

$V = \frac{\pi}{3} \cdot r^2 h$

$= \frac{\pi}{3} \left(\frac{h}{2}\right)^2 h$

$V = \frac{\pi}{3} \cdot \frac{h^3}{4}$

$\frac{dV}{dt} = 3 \frac{\pi}{12} \cdot h^2 \frac{dh}{dt}$

$= \frac{\pi}{4} (5)^2 \left(-\frac{3}{10}\right)$

$= \frac{5(-15\pi)}{10_2 (4)} = \boxed{\frac{-15\pi}{8} \ cm^3/hr}$

Work for problem 5(c)

$S = \pi r^2 \qquad r = \frac{h}{2}$

$= \pi \left(\frac{h}{2}\right)^2$

$= \frac{\pi}{4} h^2$

$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$

$\frac{dV}{dt} \propto SA$

$\frac{dV}{dt} = \pi \quad k$

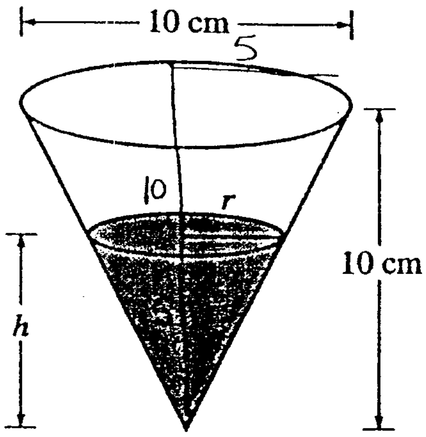
$\frac{\pi}{4} h^2 \cdot \frac{dh}{dt} = \frac{\pi}{4} h^2 \cdot k$

$\frac{dh}{dt} = k$

The constant of proportionality $= \frac{dh}{dt}$

NO CALCULATOR ALLOWED

D1



$\frac{r}{h} = \frac{5}{10}$

$5h = 10r$

$\frac{h}{2} = r$

$\frac{r}{5} = \frac{5}{10}$

$10r = 25$

$r = \frac{25}{10}$

$r = \frac{5}{2}$

Work for problem 5(a)

$\frac{dh}{dt} = -\frac{3}{10}$ cm/hr.

find: V when h = 5 cm

$V = \frac{1}{3}\pi\left(\frac{5}{2}\right)^2(5)$

$V = \frac{1}{3}\pi \frac{125}{4}$

$V = \frac{125\pi}{12}$ cm^3

NO CALCULATOR ALLOWED

D2

Work for problem 5(b)

find: $\frac{dV}{dt}$ when $h = 5$ cm

given: $\frac{dh}{dt} = -\frac{3}{10}$ cm/hr.

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$\frac{d}{dt}\left[V = \frac{1}{3}\pi \frac{h^3}{4}\right]$

$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{\pi}{4}(5)^2\left(-\frac{3}{10}\right)$

$\frac{dV}{dt} = -\frac{15\pi}{8}$ cm³/hr.

Work for problem 5(c)

$\frac{dA}{dt}\left[A = \pi\frac{h^2}{4}\right]$

$\frac{dA}{dt} = \pi\frac{h}{2}\frac{dh}{dt}$

$\frac{dA}{dt} = \pi\left(\frac{5}{2}\right)\left(-\frac{3}{10}\right)$

$\frac{dA}{dt} = \frac{-15\pi}{20}$

$\frac{dA}{dt} = -\frac{3\pi}{4}$ cm²/hr.

AP® Calculus AB
2002 Sample Student Responses

The materials included in these files are intended for use by AP teachers for course and exam preparation in the classroom; permission for any other use must be sought from the Advanced Placement Program®. Teachers may reproduce them, in whole or in part, in limited quantities, for face-to-face teaching purposes but may not mass distribute the materials, electronically or otherwise. These materials and any copies made of them may not be resold, and the copyright notices must be retained as they appear here. This permission does not apply to any third-party copyrights contained herein.

NO CALCULATOR ALLOWED

B, 1

x	−1.5	−1.0	−0.5	0	0.5	1.0	1.5
$f(x)$	−1	−4	−6	−7	−6	−4	−1
$f'(x)$	−7	−5	−3	0	3	5	7

Work for problem 6(a)

$$\int_0^{1.5}(3f'(x)+4)\,dy$$

$$\left[3F(x)+4x\right]_0^{1.5}$$

$$(3F(1.5)+4(1.5))-(3F(0)+4(0))$$

$$-3+6+21$$

$$\boxed{24}$$

Work for problem 6(b)

$f(1)=-4$

$f'(1)=5$

$$y+4=5(x-1)$$

$$y=5x-9$$

$$y(1.2)=5(1.2)-9$$

$$y(1.2)=6-9$$

$$\boxed{y(1.2)=-3}$$

The approximation is less than the actual value because on the interval, $f''(x)>0$, so $f(x)$ is therefore concave up. If this is true then tangent lines will fall below the curve, and all points on them will be below it as well.

6 6 6 6 6 6 6 6 6 6 B2

NO CALCULATOR ALLOWED

Work for problem 6(c)

f is differentiable for all real numbers, so f'(x) is differentiable for all real numbers

Mean Value Theorem says that on the interval (a, b) there exists a f'(c) such that

$f'(c) = \frac{f(b) - f(a)}{b - a}$

It applies to 2nd derivative as well.

$f''(c) = \frac{f'(.5) - f'(0)}{.5 - 0} = r$

$f''(c) = \frac{3 - 0}{.5 - 0} = r$

$f''(c) = 6 = r$

r = 6

Work for problem 6(d)

x	.5	.1	-.5	0
g'(x)	-7			∅

$g'(x) = 4x - 1$

$g'(.5) =$

$g'(0)^+ = 4x + 1 = 1$

$g'(0)^- = 4x - 1 = -1$

No because the function is not differentiable at x = 0. This violates the condition that f(x) is differentiable at all numbers, and in fact f'(0) = 0, because derivative doesn't exist at zero, function cannot be the same.

6 6 6 6 6 6 6 6 6 6

NO CALCULATOR ALLOWED

x	−1.5	−1.0	−0.5	0	0.5	1.0	1.5
$f(x)$	−1	−4	−6	−7	−6	−4	−1
$f'(x)$	−7	−5	−3	0	3	5	7

Work for problem 6(a)

$\int_0^{1.5} (3f'(x)+4)\,dx$ $\quad \int (3f'(x)+4)dx = 3f(x)+4x$

$[3f(x)+4x]\Big|_0^{1.5} = 3(-1)+6-((-7)(3))$

$= -3+6+21$

$= 3+21 = \boxed{24}$

Work for problem 6(b)

$f'(1) = 5 =$ m of tangent line

$y = 5x+b$ $\quad$ pt $(1,-4)$

$-4 = 5+b$ $\quad \boxed{y = 5x-9}$

$b = -9$

$f(1.2) \approx 5(1.2) - 9 \approx 6-9 \approx \boxed{-3}$

this is an aproximation b/c $x=1.2$ is close enough to $x=1$ that one can use tan line for $x=1$.

NO CALCULATOR ALLOWED

D_2

Work for problem 6(c)

$f''(c) = r \quad r > 0$

$f''(x) =$ slope of $f'(x)$

for $f'(x) \rightarrow (0.5, 3) \quad (0, 0)$

slope $= f''(x) = \frac{3-0}{0.5} = 6$

Mean value and Rolle's theroren

Work for problem 6(d)

$g'(x) \begin{cases} = 4x-1 & \text{for } x < 0 \\ = 4x+1 & \text{for } x \geq 0 \end{cases}$

compare to $f'(x)$ on table

no; because at $x=0$ $g'(x) = 1$
and on table $f'(0) = 0$

$1 \neq 0$

so g and f can't be the same function